Answer Keys:

Section-I

1	D	**2**	D	**3**	B	**4**	C	**5**	B	**6**	C	**7**	B
8	C	**9**	B	**10**	C	**11**	B	**12**	C	**13**	A	**14**	D
15	B	**16**	B	**17**	A	**18**	C	**19**	A	**20**	C		

Section-II

1	C	**2**	A	**3**	A	**4**	B	**5**	C	**6**	D	**7**	C
8	C	**9**	D	**10**	D	**11**	D	**12**	A	**13**	B	**14**	B
15	B	**16**	C	**17**	C	**18**	D	**19**	B	**20**	C	**21**	A
22	A	**23**	A	**24**	C	**25**	A	**26**	B	**27**	B	**28**	C
29	B	**30**	A										

Explanations:

Section-I

1. We see that our RHS exponent is 11; therefore, we set our lowest exponent to 11.
$x-4$ is certainly smaller than $x-1$, so if we let $x-4=11$ we get:
$2^{14} - 2^{11} = (2^3)(2^{11}) - 2^{11} = 8(2^{11}) - 2^{11} = 7(2^{11})$

2. $l \xrightarrow{\uparrow 20\%} 1.2l$
$b \xrightarrow{\downarrow 10\%} 0.9b$
New perimeter = $2(1.2l + 0.9b)$
Increase in perimeter = $2(0.2l - 0.1b)$
Relation between l & b is not given
$\therefore$ We can't find out the percentage increase/decrease

3. If the average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women, there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the group must have been women. Consider the following rule and its proof.

Alligation rule: The ratio that determines how to weight the averages of two or more subgroups in a weighted average also reflects the ratio of the distances from the weighted average to each subgroup's average.

4. Let x be the speed of stairway
$25 + 15x = 13 + 24x$
Or $x = \frac{4}{3}$
$\therefore 25 + 15 \times \frac{4}{3} = 45$ steps are in total

5. Rate and time are always inversely related:
AB's rate is 6/5, so in 1 hour, AB will do 5/6 of the job.
AC's rate is 3/2, so in 1 hour, AC will do 2/3 of the job.
BC's rate is 2, so in 1 hour, BC will do 1/2 of the job.

Let the number of units in the total job be a number that is a multiple of 6, 3, and 2; let's say there are 18 units in the total job.
Then, in one hour:
AB will complete 5/6*18 = 15 units;
AC will complete 2/3*18 = 12 units;
BC will complete 1/2*18 = 9 units.

Summing up:
2 As, 2 Bs, and 2Cs complete 36 units.
So, in one hour, 2 of each of the pumps will complete two jobs. Therefore, it will take 1 of each of the pumps 1 hour to complete the job.

6. It may be helpful to put the question in algebraic terms.
The tip will be equal to a constant, c, plus an amount that is proportional to the bill: kb, where k is the fraction of the bill, and b is the amount of the bill.
So the tip will be c+kb, and since we know the bill for the meal is 600/-, the tip will be c+600k.

$60 = c+700k$

$40 = c+450k$

Subtract the equations, giving you the result:

$20 = 250k \Rightarrow k = 0.08$

Then plug k back into one of the equations:

$60 = c+700(0.08) \Rightarrow c = 4$

Therefore, tip = 4+600(0.08) = 52

7. 4 men can go in five hotels in 5^4 ways.
Number of ways in which 4 men can go into different hotel

$$= 5_{P4} = \frac{5!}{(5-4)!} = 5!$$

$$\therefore \text{Required proability} = \frac{5!}{5^4} = \frac{120}{625} = \frac{24}{125}$$

8. $|A \cup B| = 40$

$|A \cup B| = |A| + |B| - |A \cap B|$

$40 = |A| + 22 - 12$

$|A| = 30$

A=30 enrolled for English & included both subjects
Number of students enrolled for English only = 30-12=18.

9. Let Mr. Vikas buys LCM (8, 5, 9) = 360 Apples of each variety.

Amount spent on the 1st variety = $\frac{360}{8}$ = 45 rs.

Amount spent on the 2nd variety = $\frac{360}{5}$ = 72 rs.

∴ Total amount spent = 45+72 = Rs.117

Now the total (360+360) = 720 Apples are sold at 9 per rupee

∴ Total revenue = $\frac{720}{9} = 80$

Hence the loss = 117-80 = 37

$\therefore \text{ Loss}\% = \frac{37}{117} \times 100 = 31.62\%$

10. Total respondents in the 21 – 30 age group = 33.

Out of them 33 – 12 = 21 like any program other than singing/dancing

$\therefore \% = \left(\frac{21}{33}\right) \times 100 = 63.63 \approx 64\%$

15. Junior/ Senior/ prior are followed by "to"

Section-II

1. $COP_{\text{ heat pump}} = (COP)_{\text{ ref}} + 1 \therefore (COP)_{ref} = 5 - 1 = 4$

Now, $(COP)_{ref} = \frac{RE}{W}$; $RE = 1 \times 4 = 4kW$

3. According to shear stress theory,

$$\tau_y = \frac{\sigma_y}{2} = \frac{250}{2} = 125\,MPa$$

$$FOS = 2.5$$

According to the information given in this problem,
design should be done on the basis of Rivet failure
Load on each rivet = 2.5 kN
Since, two plate are connected by Rivet, only single shear
should be taken into account.

$$\frac{2.5 \times 10^3}{\frac{\pi}{4} d^2} \leq \frac{125}{2.5}$$

$$d \geq 7.9788\,mm \simeq 8\,mm$$

4. $V = 0.05 m^3; V_f = m_f \nu_g = 8 \times 0.0012512 = 0.0100096$

$V = V_f + V_g \Rightarrow 0.05 = 0.0100096 + V_g \Rightarrow V_g = 0.03999$

$$V_g = m_g \nu_g \Rightarrow m_g = \frac{V_g}{\nu_g} = \frac{0.03999}{0.05013} = 0.797725$$

$$x = \frac{m_g}{m_g + m_f} = \frac{0.797725}{0.797725 + 8} = 0.0906$$

$h = h_f + x h_{fg} = 1085.36 + 0.906(1716.2) = 1240.96 \text{ kJ/kg}$

5. Expectation of $X = E(x)$

$$= \text{Average} = \text{mean} = \frac{1}{n}\sum_{1}^{n} X_i = \sum_{i=1}^{n} P_i x_i$$

Where P_i is probability of occurrence of x_i

Let $q = 1 - p$

Then probability of success in 1st, 2nd and 3rd trials are p, qp, q^2p

$$\therefore E(x) = 0 \cdot p + 1 \cdot qp + 2 \cdot q^2 p + .. = qp\left[1 + 2q + 3q^2 + ... + nq^{n+1} + ...\right]$$

$$= \frac{qp}{(1-q)^2} = \frac{qp}{p^2} = \frac{q}{p}$$

6. In Scotch yoke mechanism, the slides is sliding in a reciprocating body. Radial relative velocity of slider is zero, hence, Coriolis acceleration is zero.

7. $A = 1963.5 \text{ mm}^2, L = 1500\text{mm}, P = 200\text{N}, h = 25\text{mm}$

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2EAh}{P \times L}}\right) = 26.16 \text{ N/mm}^2$$

8. For an ideal gas, $\left(\frac{\partial T}{\partial S}\right)_p = \frac{T}{C_P}$

$$\Rightarrow \left(\frac{\partial T}{\partial S}\right)_p = \frac{T}{C_v}$$

As $C_P > C_V \Rightarrow \left(\frac{\partial T}{\partial S}\right)_P < \left(\frac{\partial T}{\partial S}\right)_V$

9. $$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

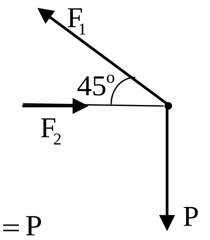
$$N_2 = \frac{T_1}{T_2} \times N_1$$

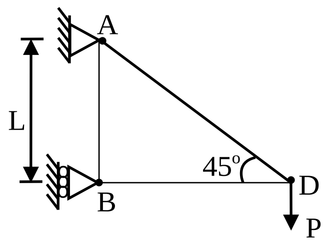
$$N_2 = \frac{20}{34} \times 1000$$

$N_2 = 588.23$ rpm

10. Two member are connected at joint C & it has no external load or any support. So, according to zero force member theorem, force in both member (i.e., AC & CD) are zero. Therefore, that can be removed from the structure.

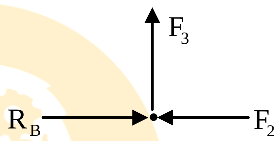
Apply method of joint at 'D',





$$F_1 \text{Sin} 45 = P$$
$$F_1 = P\sqrt{2}$$
$$F_2 = F_1 \text{Cos} 45$$
$$F_2 = P$$

Apply method of joint at Joint 'B'



$$\sum F_y = 0$$
$$F_3 = 0$$

Strain Energy for AB member,

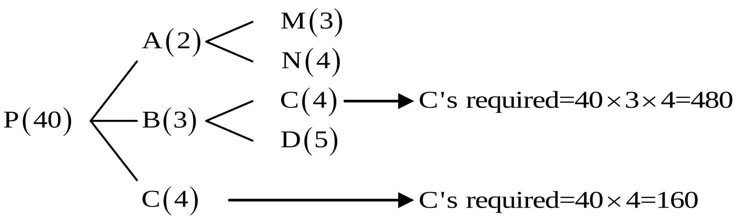
$$U = \frac{F_3^2 L}{2AE} = 0$$

11. $$I = \iint_R xy(x+y)\,dydx = \int_{x=0}^{1}\int_{y=x^2}^{x}\left(x^2y + xy^2\right)dydx$$

$$= \int_{x=0}^{1}\left\{x^2\left(\frac{y^2}{2}\right)_{y=x^2}^{x} + x\left[\frac{y^3}{3}\right]_{y=x^2}^{x}\right\}dx$$

$$= \int_{x=0}^{1}\left[\frac{x^4}{2} - \frac{x^6}{2} + \frac{x^4}{3} - \frac{x^7}{3}\right]dx = \frac{x^5}{10} - \frac{x^7}{14} + \frac{x^5}{15} - \frac{x^8}{24}\Bigg|_{x=0}^{1} = \frac{3}{56}$$

12.



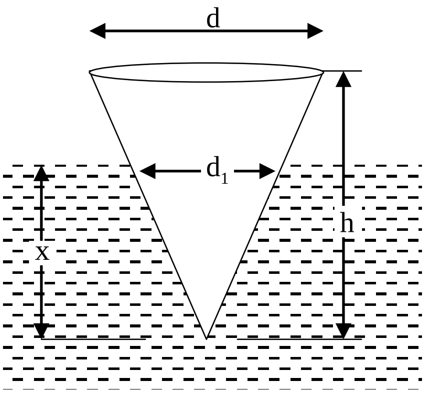
Total Number of C's required = 480+160 = 640

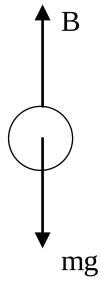
Given number of B's on hand = 50 i.e. No. of C's on hand = 50 x 4 = 200;

Since each B contains 4 C's

Therefore number of C's needed = 640 – 200 = 440

13. Let diameter = d, height = h, specific gravity = s
And x be the underwater height of cone.





$$\Rightarrow \frac{1}{3}\frac{\pi}{4}d^2\ hs = \frac{1}{3}\frac{\pi d_1^2}{4}x \quad (\text{w.r.t volumes})$$

$$\therefore \frac{d_1^2}{d^2} = \frac{hs}{x}$$

But $\frac{d_1}{d} = \frac{x}{h}\ \therefore \frac{x^2}{h^2} = \frac{hs}{x} \Rightarrow x = hs^{1/3}$

(By Geometry)

C.G. of cone is at a depth of $\frac{1}{4}^{th}$ of height of cone below the base.

$$\overline{BG} = \frac{3h}{4} - \frac{3x}{4} = \frac{3}{4}\left(h - hs^{1/3}\right)$$

$$\overline{BM} = \frac{I}{V}, I = \frac{\pi}{64}d_1^4\ \&\ V = \frac{1}{3}\frac{\pi d_1^2}{4}x$$

$$\overline{BM} = \frac{3d_1^2}{16x} = \frac{3}{16}\frac{d^2x}{h^2}$$

$$= \frac{3}{16}\frac{s^{1/3}d^2}{h}$$

$$\text{So, } \overline{BM} = \frac{3}{16} \times \frac{(0.027)^{1/3}\ 2^2}{4} = 0.5625$$

$$\overline{BG} = \frac{3}{4} \times \frac{(0.027)^{1/3}\ 2^2}{4} = 0.5625$$

$$\overline{BG} = \frac{3}{4} \times 4(1 - 0.3) = 2.1$$

Here $\overline{BM} < \overline{BG}$

So, the cone is in unstable in water.

14. $$Q = \frac{T_i - T_o}{\dfrac{r_o - r_i}{4\pi r_o r_i k}} = \frac{250 - 100}{\dfrac{0.15 - 0.10}{4 \times \pi \times 0.15 \times 0.1 \times 50}} = 28.27\ \text{kW}$$

15. Given,

$$A_2 = 6.4\text{cm}^2$$
$$Q = 900\text{cm}^3/\text{s}$$
$$Q = A_2V_2 = A_2\sqrt{2gh_c}$$
$$900 = 6.4\sqrt{2\times 981\times h_c}$$
$$\therefore h_c = 10.08\text{ cm}$$
$$\frac{A_2}{A_3} = \sqrt{\frac{h_t}{h_c}} \quad \text{(using continuty equation)}$$
$$A_3 = A_2\sqrt{\frac{h_c}{h_t}}$$
$$= 6.4\sqrt{\frac{10.08}{25+10.08}} = 3.43\text{ cm}^2$$

16. Applying work energy equation to the entire rope for an infinitesimal movement

$$\text{pdx} = d\left(\frac{1}{2}\rho x v^2\right) + d(\rho g x\times x)$$
$$\text{pdx} = \frac{1}{2}\rho v^2 dx + \rho g x dx$$
$$p = \frac{1}{2}\rho v^2 + \rho g x$$

17. $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)};$$
$$x_n - \frac{(3x_n{}^2 + 2x_n + 1)}{6x_n + 2} = \frac{3x_n{}^2 - 1}{6x_n + 2}$$

18. $$\omega_1 = 10\text{rad/sec};\ \omega_2 = \frac{300\times 2\pi}{60} = 10\pi\text{rad/s}$$
$$y = \frac{e}{1-\left(\frac{\omega_1}{\omega_2}\right)^2} = \frac{2}{1-\left(\frac{10}{10\pi}\right)^2} = 2.25\text{mm}$$

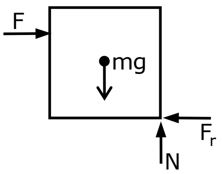
So, radius of whirl = y + r = 4.25 mm

19. Humidity ratio of air $= 0.622\left(\frac{P_V}{P - P_V}\right)$

$$= 0.622\left(\frac{2.344}{101.325 - 2.344}\right) = 0.01472$$

Relative humidity of moist air sample $= \frac{P_V}{P_S} = \frac{2.344}{3.36}\times 100 = 69.76\%$

20. At the time of increasing force



Check for toppling

$N = 20N \quad F = xN \ (0 < x \le 6.5)$

$F_r = F \quad \left[\text{as } F_r|_{max} = \mu N = 10N\right]$

Moment about CG

$x \times \frac{50}{4} + x \times 25 - 20 \times 12 = 0$

$\Rightarrow x = \frac{240 \times 4}{150} = 6.4$

That means when the force reaches 6.4N the object will topple first.

Check for equilibrium in new position,

$F = F_r = 6.5N$

$N = mg = 20N$

To avoid toppling minimum length of base should be x

$\frac{3}{4} h \times 6.5 - 20 \times \frac{x}{2} \le 0$

$x \ge 0.75 \times 6.5 \times 0.1 \times 50$

$x \ge 24.375 mm$

Hence N will reach edge and goes further. That means sign of toppling. And hence the body will topple

21. $\int_0^\infty \int_y^\infty x e^{-\frac{x}{y^2}} dx dy = \int_{y=0}^{\infty} \left\{ \int_{x=y} x e^{-x^2/y} dx \right\} dy$

$$= \int_{y=0}^{\infty} \left\{ \int_{x=y}^{\infty} \left[\frac{-y}{2} e^{\frac{-x^2}{y}} \left(\frac{-2x}{y} dx \right) \right] \right\} dy = \int_0^\infty \left\{ \frac{-y}{2} \left[e^{\frac{-x^2}{y}} \right]_{x=y}^{\infty} \right\} dy = \int_0^\infty \frac{y}{2} e^{-y} dy$$

$$= \frac{1}{2} \left\{ \left[\frac{y e^{-y}}{-1} \right]_0^\infty - \int_0^\infty \left(\frac{e^{-y}}{-1} \right) dy \right\} = \frac{1}{2} \left[\frac{e^{-y}}{-1} \right]_0^\infty = \frac{1}{2} = 0.5$$

22. Energy balance $\alpha + \rho + \tau = 1$

$\frac{Q_a}{Q_o} + \rho + \tau = 1$

$\frac{Q_a}{Q_o} + 0.032 + 0.55 = 1$

Incident flux then $Q_o = \frac{Q_a}{1 - 0.032 - 0.55}$

$= \frac{95}{0.418} = 227.27 \ W/m^2.$

23. Maximum Mass Flow Rate ⇒ Maximum Volume Flow Rate

$$\dot{m}_{max} = \rho A \dot{V}_{max}$$

$$\dot{V}_{max} \Rightarrow \frac{d\dot{V}}{dS} = 0 \Rightarrow S = 1.5\frac{km}{min}$$

$$\frac{d^2V}{dS^2} < 0, \text{So } S = 1.5\frac{km}{min}\left(\dot{V} \text{ maximum}\right)$$

$$\therefore \dot{V}_{max} = 3S - S^2 = 3\times1.5 - 1.5^2$$

$$= 4.5 - 2.25$$

$$= 2.25 \text{ LPM}$$

$$\therefore \dot{m}_{max} = 1200\times4\times10^{-4}\times2.25\times10^{-3}\frac{kg}{min} = 1.08\frac{grams}{min}.$$

24. Approach allowance is

$$= \frac{1}{2}\left(D - \sqrt{D^2 - b^2}\right)$$

$$= \frac{1}{2}\left(600 - \sqrt{600^2 - 40^2}\right)$$

$$= 0.67\text{mm}$$

Approach allowance =over run allowance = 0.67 mm

Table travel L = $L_1+L_2+L_3$

= 300+0.67+0.67

= 301.34 mm

Table feed/min f = $f_t \times Z \times N$

= 0.1×6×1000

= 600 mm/min

Machining time = (L × n)/f

= (301.34×1)/600

= 0.502 min

25. $dy = \left(\frac{2}{x} - 4x^2\right)dx \Rightarrow y = 2\ln x - \frac{4x^3}{3} + c$

26. $\text{Total load} = 2P + W_b = 2\times2250 + 90 = 4950\,\text{kg}$

$$\text{Direct shear} = \sigma_s = \frac{4950}{\frac{\pi}{4}d^2\times4}\frac{kgf}{cm^2} = \frac{1474}{d^2}\frac{kgf}{cm^2}$$

Assume load per unit distance as ‘L’.

Load on each bolt at BB $= L\times7.5$

Load on each bolt at AA = $L\times52.5$

Taking moment about pivot point of rocking

$$\left(2\times L\times 52.5^2\right)+\left(2\times L\times 7.5^2\right)=90\times 37.5+2\times 2250\times 45$$

$\Rightarrow L = 36.6 \text{ kg / cm}$

So load at AA (maximum) $= L\times 52.5 \text{ kg} = 1921.5 \text{kg}$

Maximum load on bolts at AA and tensile stress in each bolt at section AA

$$S_t = \frac{1921.5}{0.7854 d_c^2} = \frac{2446.5}{d_c^2}$$

Principal tensile stress in each bolt at AA

$$= \frac{S_t}{2} \pm \sqrt{\left(\frac{S_t}{2}\right)^2 + S_s^2}$$

$$= \frac{1223.5}{d_c^2} \pm \sqrt{\left(\frac{1223.5}{d_c^2}\right)^2 + \left(\frac{1461.04}{d^2}\right)^2}$$

As $d = \frac{d_c}{0.84}$

Max. principle tensile stress,

$$= \frac{1223.25}{d_c^2} + \sqrt{\left(\frac{1223.25}{d_c^2}\right)^2 + \left(\frac{1461.04\times 0.84}{d_c^2}\right)^2}$$

$$= \frac{2956.03}{d_c^2}\frac{\text{kgf}}{\text{cm}^2}$$

$$\text{principal shear stress} = \sqrt{\frac{(1223.25)^2 + (1461.04\times 0.84)^2}{d_c^4}}$$

$$= \frac{1732.78}{d_c^2}\frac{\text{kgf}}{\text{cm}^2}$$

But, allowable shear stress $= 350 \frac{\text{kgf}}{\text{cm}^2}$

$$\therefore d_c = \sqrt{\frac{1732.78}{350}} = 2.225 \text{cm}$$

So, full diameter $= \frac{22.25}{0.84} = 26.48$ mm

So, we can take bolt diameter greater than $26.5 = 28 \text{mm}$

27. $L_{DA} = \sqrt{15^2 + x^2}\,; L_{CD} = 15 - y, L = L_{DA} + L_{CD}.$

$30 = \sqrt{15^2 + x^2} + (15 - y),$

$y = \sqrt{225 + x^2} - 15$

$$V_S = \frac{dy}{dt} = \frac{1}{2}\frac{2x}{\sqrt{225+x^2}}\frac{dx}{dt} = \frac{x}{\sqrt{225+x^2}}V_A$$

at $y = 10\text{m},\ x = 20\text{m},\ V_A = 0.5, V_S = 0.4 \text{m / sec}.$

28. $R = 9000/\text{year};\ C_0 = 9000/\text{setup};\ C_u = 500$

$\ell = 0.2;\ K = 200/\text{day} = 73000/\text{year};$

$$EOQ = \sqrt{\frac{2RC_o}{\ell C_u} \times \frac{K}{K-R}} = \sqrt{\frac{2 \times 73000 \times 9000 \times 9000}{0.2 \times 500 \times 64000}} = 1359.3$$

$$\frac{Q'}{Q} = \frac{K-R}{K} \Rightarrow Q' = \frac{64000}{73000} \times 1359.3 = 1191.75 \cong 1192$$

29. Power developed = 2850 kW

Head, $H = 5.2m$

Height of draft inlet tube above tailrace level, $H_s = 1.8m$

Reading of the gauge $= -5.2m$

Draft tube efficiency, $\eta_d = 75\%$

Overall efficiency of the turbine, η_0

$$\frac{P_2}{w} = \frac{P_a}{w} - H_s - \left(\frac{V_2^2 - V_3^2}{2g} - h_f\right)$$

$$-5.2 = 0 - 1.8 - \left(\frac{V_2^2 - V_3^2}{2g}\right), \text{neglect } h_f$$

$$\frac{V_2^2 - V_3^2}{2g} = 3.4$$

$$\text{or, } \eta_d = \frac{(V_2^2 - V_3^2)/2g}{(V_2^2/2g)}, \text{ or } 0.75 = \frac{3.4}{V_2^2/2g} \text{ or } \frac{V_2^2}{2g} = \frac{3.4}{0.75} = 4.533$$

$$\therefore V_2 = \sqrt{4.533 \times 2g} = \sqrt{4.533 \times 2 \times 9.81} = 9.43 m/s$$

$$\text{Discharge } Q = \frac{\pi}{4} \times 3^2 \times 9.43 = 66.65 m^3/s$$

$$\therefore \text{ Overall efficiency, } \eta_0 = \frac{\text{Power developed}}{\text{Water power}} = \frac{2890}{WQH}$$

$$= \frac{2850}{9.81 \times 66.65 \times 5.2} = 0.8382 \text{ or } 83.82\%$$

30. Total length of stroke = 610 + 80 + 80 = 770mm

Cutting time = $\frac{770 \times 60}{1000 \times 10}$ = 4.62sec

$\frac{\text{Return time}}{\text{cutting time}} = \frac{1}{4} \Rightarrow$ Return time = 4.62 x $\frac{1}{4}$ = 1.155 sec

Time to complete one double stroke = 4.62 + 1.155= 5.77sec

Total number of double strokes = $\frac{920}{3}$ =306

Total time required to complete the cut = $\frac{5.77 \times 306}{60}$ =29.45min