Answer Keys:

Section-I

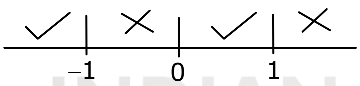
1	B	**2**	D	**3**	C	**4**	B	**5**	A	**6**	A	**7**	D
8	A	**9**	D	**10**	C	**11**	D	**12**	B	**13**	D	**14**	D
15	D	**16**	D	**17**	A	**18**	A	**19**	D	**20**	B		

Section-II

1	B	**2**	C	**3**	B	**4**	A	**5**	A	**6**	C	**7**	B
8	B	**9**	A	**10**	C	**11**	D	**12**	C	**13**	C	**14**	A
15	C	**16**	B	**17**	C	**18**	C	**19**	B	**20**	B	**21**	B
22	C	**23**	A	**24**	A	**25**	D	**26**	C	**27**	A	**28**	C
29	C	**30**	C										

Explanations:

Section-I

1. 

For $x > 1$, $(x-1)x(x+1) \le 0$ condition is not satisfied

For $0 \le x \le 1$, $(x-1)x(x+1) \le 0$ condition is satisfied

$-1 < x < 0$, $(x-1)x(x+1) \le 0$ condition is not satisfied

$x \le -1$, $(x-1)x(x+1) \le 0$ condition is satisfied

2. Since 70% of the employees received bonuses of at least 10,000, 30% of the employees received bonuses of less than 10,000. We know that 60 employees received bonuses of less than 10,000. If E is the number of employees, we can set up the following equation:
.30E = 60, E = 200
40% of the employees received bonuses of at least 50,000. Thus, (40% × 200) or 80 employees received bonuses of at least 50,000. 20% of the employees received bonuses of at least 1,00,000. Thus, (20% × 200) or 40 employees received bonuses of at least 1,00,000. If 80 employees received at least 50,000, and 40 employees received at least 1,00,000, then (80 - 40) or 40 employees received bonuses of at least 50,000 but less than 1,00,000.

3. $P\left(1+\frac{r}{100}\right)^5 = 3P, \quad P\left(1+\frac{r}{100}\right)^{10} = 9P$

4. Average = sum of terms / number of terms. In this question, we can apply the formula to the difference between the average we got and the average we were supposed to get:

$1.8 = \frac{\text{extra sum of terms}}{10} \Rightarrow 18 = \text{extra sum of terms}$

So, 'ut' is 18 more than 'tu'

'ut'= $10u+1\times t=10u+t$

'tu'=$10t+1\times u=10t+u$

'ut' - 'tu' = 9 (u-t)

i.e. 18 = 9 (u-t)

Therefore, u-t=2

5. Let Tap A fills 1 liter in a minute.
Then Tap B fills 2 liters per min (that is why Tap A is taking double time)
Together, they will fill 3 lt per min.
In 6 hours, they will fill 18liters (which is capacity of tank).
To fill 18liters (full tank), Tap A will take 18 hours

6. When there is a loss at 10% $\rightarrow$ 160 = 90% of CP_2

$\therefore C.P_2 = 177.37$

When there is a profit of 10% $\rightarrow$ 160 = 110% of CP_1

$\therefore C.P_1 = 145.45$

Total $C.P = 177.77 + 145.45 = 323.23$

$\text{Loss} = 3.23$

7. Let the ages of children is $x, (x+3), (x+6), (x+9) \& (x+12)$ yrs.

Then $x + (x+3) + (x+6) + (x+9) + (x+12) = 50$

$5x + 30 = 50 \Rightarrow x = 4$

8. $x = \frac{90}{360} \times 45,000 = 11,250\text{rs};$

$y = \frac{120}{360} \times 45,000 = 15,000\text{rs}$

$z = \frac{150}{360} \times 45,000 = 18,750\text{rs};$

Hence in 1997 the costs are:

$x = 11,250 \times 1.1 = \text{Rs. }12375$

$y = 15,000 \times 1.3 = \text{Rs. }19500$

$z = 18,750 \times 1.2 = \text{Rs. }22500$

Total cost = 12375 + 19500 + 22500 = 54375

9. $1-9 \quad 9\times 1\text{digits} = 9$

$10-99 \quad 90\times 2\text{digits} = 180$

$100-999 \quad 900\times 3\text{digits} = 2700$

2889

2777^{th} digit is of a 3 digit number

$2889-2777=112=37\times 3+1$

From 999, 37 numbers behind is 962. Its second digit is required answer. So answer is 6.

10. i. $\frac{\text{B in 2011}}{\text{C in 2012}} = \frac{15}{55} = \frac{3}{11}$

ii. Average $= \frac{10+15+40+40}{4} = \frac{105}{4} = 26.25$

iii. Percentage increase in C $= \frac{30\text{-}15}{15}\times 100\% = 100\%$

Section-II: Technical

1. $P+F=C+2 \Rightarrow 2+F=1+2 \Rightarrow F=1$

2. In general, No. of teeth = 40

No. of divisions on crank $= \frac{40}{\text{No. of divisions to be made}} = \frac{40}{23} = 1\frac{17}{23}$

Hence plate (2) will be used for indexing which have 23 holes circle

Hence option (C) is correct

3. Uniform pressure applied. $\Delta V = 0$,

$\left(\text{Hydrostatic pressure, volumetric strain} = \frac{\Delta V}{V} = 0\right)$

4. The two balls drawn may be both green, one green and one red or both red.

In these cases, the man receives 40paise, 30paise and 20paise respectively.

Let X be the amount the man receives. Then

$P[x=40] = P[\text{both green}] = \frac{^3C_2}{^5C_2} = 0.3$

x	40	30	20		
P(x)	0.3	0.6	0.1		

$P[x=30] = P[\text{one green one red}] = \frac{3C_1\times 2C_1}{5C_2} = 0.6$

$P[x=20] = P[\text{both red}] = \frac{2C_2}{5C_2} = 0.1$

$\therefore$ Probability distribution of x is

$E(x) = \Sigma x P(x) = 40\times 0.3 + 30\times 0.6 + 20\times 0.1 = 32\text{paise}$

6. $\Psi_1 = 2x^2 - 3y^2$

$-u = \frac{d\Psi}{dy}; \quad v = \frac{d\Psi}{dx}$

$u = 6y, \; v = 4x$

$a_x = u\frac{du}{dx} + v\frac{du}{dy}$

$\Rightarrow a_x = 4x \times 6 = 24x$

$a_y = u\frac{dv}{dx} + v\frac{dv}{dy}$

$= 6y \times 4$

$= 24y$

acceleration vector $= 24xi + 24yj$

8. Since CI is brittle, fails along a plane of 45^o

9. $\frac{\partial f}{\partial x} = nx^{n-1}; \; \frac{\partial f}{\partial y} = ny^{n-1}; \; \frac{\partial f}{\partial z} = nz^{n-1}$

$\Rightarrow \nabla f = n\left[x^{n-1}i + y^{n-1}j + z^{n-1}k\right]$

Since r = xi+yj+zk,

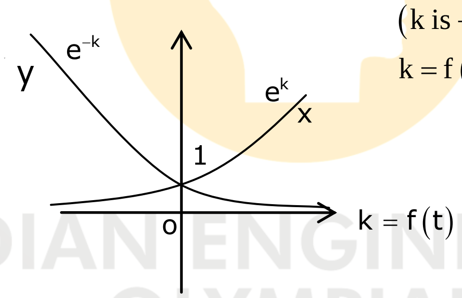
$\nabla f \cdot r = \left(nx^{n-1}x\right) + \left(ny^{n-1}y\right) + \left(nz^{n-1}z\right) = n\left(x^n + y^n + z^n\right) = nf$

12. $X = \mu(t) = \exp(+\text{ve number}) = e^k \; (\because T^4, T^2, 8 \text{ are } +\text{ve})$

$Y = \mu(t) = \exp(-\text{ve number}) = e^{-k} \; (\because -T^6, -T^8, -23 \text{ are } -\text{ve})$

$(k \text{ is } +\text{ve})$

$k = f(T)$



From the above graph

For X as Temperature is increasing Viscosity also increasing so it is a gas

For Y as Temperature decreasing Viscosity is decreasing so it is a liquid

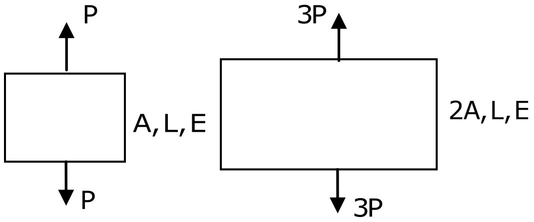
14. $\underset{x\to\infty}{Lt}\left[\frac{x^2+5x+3}{x^2+x+2}\right]^x$

$= \underset{x\to\infty}{Lt}\left(1+\frac{4x+1}{x^2+x+2}\right)^x$

$= \underset{x\to\infty}{Lt}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{x(4x+1)}{x^2+x+2}}$

$= e^{\underset{x\to\infty}{Lt}\frac{x(4x+1)}{x^2+4x+2}} = e^{\underset{x\to\infty}{Lt}\frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{1}{x^2}}} = e^4$

15. $\Delta\ell = \frac{3P \times L}{2A \times E} + \frac{PL}{AE} = 2.5\frac{PL}{AE}$



16. $P_1V_1^{1.3} = P_2V_2^{1.3} \Rightarrow P_2 = 0.5 \times \left(\frac{0.02}{0.05}\right)^{1.3}$

$= 0.152 \text{ MPa}$

work done in adiabatic process $= \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$= \frac{(5 \times 10^5 \times 0.02) - (1.52 \times 10^2 \times 0.05)}{1.3 - 1} = 8\text{kJ}$

17. $\frac{\partial V}{\partial S} = \frac{44.5 - 4.5}{0.4} = 100$

Convective Acceleration $= V\frac{\partial V}{\partial S}$

acceleration at beginning $(a_b) = 4.5 \times 100 = 450 \, m/s^2$

acceleration at end $(a_e) = 44.5 \times 100 = 4450 \, m/s^2$

So $(a_e - a_b) = 4450 - 450 = 4000 \, \frac{m}{s^2}$

18. For 'A' potential energy = mgh = 1xgx20 J

When if rolls down a frictionless slope (number loss of energy)

$\Rightarrow (P.E)_A = (K.E)_A$

$\Rightarrow 1 \times g \times 20 = \frac{1}{2} m \, V_A^2$ which gives $V_A = 20 \, m/sec$

As per law of conservation of linear momentum

$m_A V_A = m_B V_B$

$\therefore V_B = V_A = 20 \, m/sec \; (\text{As } m_A = m_B)$

Sphere 'B' moving with 20 m/sec velocity but due to friction on the floor (surface) its energy reduces as it reaches spring by $\mu mg x$

$\therefore \text{Energy} = \frac{1}{2} m_B V_B^2 - \mu m_B gx = \frac{1}{2} \times 1 \times 20^2 - 0.2 \times 1 \times 10 \times 2 = 196\text{J}$

This energy equals spring compression energy

$\therefore 196 = \frac{1}{2} k \delta^2 \Rightarrow k = \frac{2 \times 196}{(0.1)^2} = 39.2 \, N/mm$

19. $\int_c Mdx + Ndy = \iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right)dxdy, M = xy + y^2;\ N = x^2$

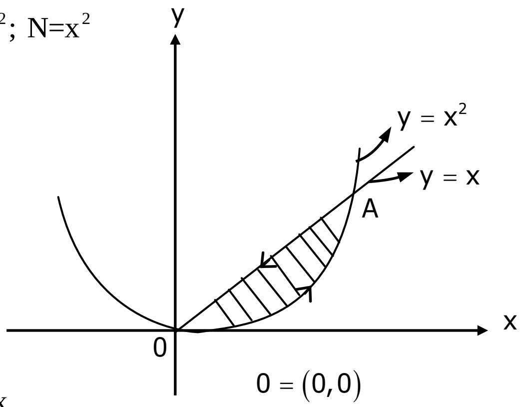
$$\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y} = 2x - (x + 2y) = x - 2y$$

$$\iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right)dxdy$$

$$= \int_{x=0}^{1}\int_{y=x^2}^{x} (x - 2y)dydx = \int_{x=0}^{1}\left[xy - y^2\right]_{y=x^2}^{x} dx$$

$$= \int_{x=0}^{1}\left[\left(x^2 - x^2\right) - \left(x^3 - x^4\right)\right]dx = \int_{x=0}^{1}\left(x^4 - x^3\right)dx$$

$$= \frac{1}{5} - \frac{1}{4} = \frac{-1}{20}$$



$0 = (0,0)$
$A = (1,1)$

y varies x^2 to x
x varies 0 to 1

21. unavailable energy $= 7200 - 4800 = 2400$ kJ / kg,

$$T_o \times \Delta s = 2400 \Rightarrow T_o = \frac{2400}{8} = 300\,K$$

22. Mobility $= 3(n-1) - 2j - h$

$$= 3(5-1) - 2\times 5 - 1$$

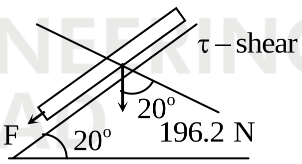
$$= 12 - 10 - 1 = 1$$

23.

$$F = 196.2 \times \sin 20 = 67.10\,N$$

$$\tau = \frac{F}{A} = \frac{67.10}{0.20 \times 0.20} = 1677.60\ \frac{N}{m^2}$$

$$\tau = \mu\frac{du}{dy} : \mu = 2.158 \times 10^{-3}\ \frac{Ns}{m^2}$$



$$dy = 0.025\,mm = 0.025 \times 10^{-3}\,m$$

$$1677.60 = 2.158 \times 10^{-3}\ \frac{du}{0.025 \times 10^{-3}}$$

$$V = du = 19.43\,m/s$$

24. We have, $e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3}.....$

$$\sin\frac{1}{z} = \frac{1}{z} - \frac{1}{3!z^3} + ...$$

There is an essential singularity at z = 0

The residue at z = 0 is coefficient of $\frac{1}{z}$ in Laurent series of integrand, which is 1

So $\oint_{|z|=1} e^{1/z}\sin\frac{1}{z}dz = 2\pi i$

25. $\varepsilon_1 + \varepsilon_2$

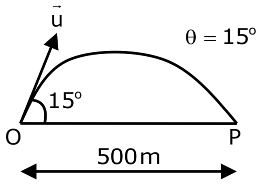
True strain is equal to the sum of incremental strains

26. Range $C = \dfrac{u^2 \sin 2\theta}{g}$

$$500 = \frac{u^2 \sin 30}{10}$$

$\therefore \quad u = 100 \text{ m/sec}$

Person started instantaneously at 10 m/sec to fire first bomb hitting ground i.e., distance $= 10 \times \text{Time of flight}$

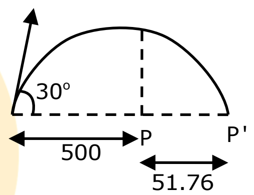


$$\text{Time of flight} = \frac{2u\sin\theta}{g} = \frac{2\times100}{10}\sin 15$$

$$= 5.176 \text{ seconds}$$

$\therefore \text{Distance} = 10 \times 5.176 = 51.76 \text{m}$

So New Range $= 500 + 51.76 \text{m} = 551.76 \text{m}$



$$\text{New range} = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 60^\circ}{10}$$

$$\therefore \frac{u^2 \sin 60}{10} = 551.76$$

$\Rightarrow u = 80 \text{m/sec}$

27. Load at gudgeon pin is reversed in direction when Piston effort is zero

i.e., $F = F_g - F_I = 0$

$\Rightarrow F_g = F_I$

$F_g = 6000$ N,

$$F_I = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) \left[\text{Where, } n = \frac{l}{r} = 5\right]$$

$$= 3\times\left(60\times10^{-3}\right)\times\omega^2\left(\cos 30^\circ + \frac{\cos 60}{5}\right)$$

$\omega = 185.76$ rad/s.

28. Number of links n = 10

Number of binary joints j = 14

DOF = 3(10-1)-2(14) = -1

But this is an exception case where parallelogram linkage is used.

$\therefore$ DOF = 1

29. $P_1V_1^{1.5} = P_2V_2^{1.5}$

$$P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^{1.5} = 3\times\left(\frac{0.1}{0.2}\right)^{1.5} = 1.06\,\text{bar}$$

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{(3\times10^2\times0.1)-(1.06\times10^2\times0.2)}{1.5-1}$$

$$= 17.6\,\text{kJ}$$

Net heat transfer for the process, $Q = (U_2 - U_1) + W$

$$= m(U_2 - U_1) + W$$

$$= 4(-4.6) + 17.6$$

$$= -0.8\,\text{kJ}$$

30. $$a_x = \frac{\partial U}{\partial t} + u\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} + w\frac{\partial U}{\partial z} = 0 + \left(\frac{U_0x}{L}\right)\left(\frac{U_0}{L}\right) = \frac{U_0^{\,2}x}{L^2}$$

$$a_y = 0 + 0 + \left(\frac{-U_oy}{L}\right)\left(\frac{-U_o}{L}\right) = \frac{U_o^{\,2}y}{L^2}$$

$$a = a_xi + a_yi = \frac{U_o^{\,2}}{L^2}(xi + yj) = \frac{U_o^{\,2}}{L^2}r$$