Answer Keys:

Section-I

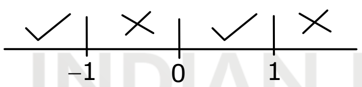
1	B	**2**	D	**3**	C	**4**	B	**5**	A	**6**	A	**7**	D
8	A	**9**	D	**10**	C	**11**	D	**12**	B	**13**	D	**14**	D
15	D	**16**	D	**17**	A	**18**	A	**19**	D	**20**	B		

Section-II

1	A	**2**	D	**3**	A	**4**	B	**5**	A	**6**	A	**7**	A
8	B	**9**	C	**10**	B	**11**	C	**12**	A	**13**	A	**14**	B
15	D	**16**	B	**17**	C	**18**	A	**19**	A	**20**	A	**21**	A
22	C	**23**	D	**24**	B	**25**	A	**26**	B	**27**	B	**28**	A
29	D	**30**	C										

Explanations:

Section-I

1. 

For $x > 1$, $(x-1)x(x+1) \le 0$ condition is not satisfied

For $0 \le x \le 1$, $(x-1)x(x+1) \le 0$ condition is satisfied

$-1 < x < 0$, $(x-1)x(x+1) \le 0$ condition is not satisfied

$x \le -1$, $(x-1)x(x+1) \le 0$ condition is satisfied

2. Since 70% of the employees received bonuses of at least 10,000, 30% of the employees received bonuses of less than 10,000. We know that 60 employees received bonuses of less than 10,000. If E is the number of employees, we can set up the following equation: .30E = 60, E = 200
40% of the employees received bonuses of at least 50,000. Thus, (40% × 200) or 80 employees received bonuses of at least 50,000. 20% of the employees received bonuses of at least 1,00,000. Thus, (20% × 200) or 40 employees received bonuses of at least 1,00,000. If 80 employees received at least 50,000, and 40 employees received at least 1,00,000, then (80 - 40) or 40 employees received bonuses of at least 50,000 but less than 1,00,000.

3. $P\left(1+\frac{r}{100}\right)^5 = 3P, \quad P\left(1+\frac{r}{100}\right)^{10} = 9P$

4. Average = sum of terms / number of terms. In this question, we can apply the formula to the difference between the average we got and the average we were supposed to get:

$1.8 = \frac{\text{extra sum of terms}}{10} \Rightarrow 18 = \text{extra sum of terms}$

So, 'ut' is 18 more than 'tu'

'ut'= 10u+1× t=10u+t

'tu'=10t+1× u=10t+u

'ut' - 'tu' = 9 (u-t)

i.e. 18 = 9 (u-t)

Therefore, u-t=2

5. Let Tap A fills 1 liter in a minute.
Then Tap B fills 2 liters per min (that is why Tap A is taking double time)
Together, they will fill 3 lt per min.
In 6 hours, they will fill 18liters (which is capacity of tank).
To fill 18liters (full tank), Tap A will take 18 hours

6. When there is a loss at 10% → 160 = 90% of CP_2

$\therefore C.P_2 = 177.37$

When there is a profit of 10% → 160 = 110% of CP_1

$\therefore C.P_1 = 145.45$

$\text{Total C.P} = 177.77 + 145.45 = 323.23$

$\text{Loss} = 3.23$

7. Let the ages of children is x, $(x+3),(x+6),(x+9) \& (x+12)$ yrs.

Then $x + (x+3) + (x+6) + (x+9) + (x+12) = 50$

$5x + 30 = 50 \Rightarrow x = 4$

8. $x = \frac{90}{360} \times 45{,}000 = 11{,}250 \text{ rs;}$

$y = \frac{120}{360} \times 45{,}000 = 15{,}000 \text{ rs}$

$z = \frac{150}{360} \times 45{,}000 = 18{,}750 \text{ rs;}$

Hence in 1997 the costs are:

$x = 11{,}250 \times 1.1 = \text{Rs. } 12375$

$y = 15{,}000 \times 1.3 = \text{Rs. } 19500$

$z = 18{,}750 \times 1.2 = \text{Rs. } 22500$

$\text{Total cost} = 12375 + 19500 + 22500 = 54375$

9. $1-9 \quad 9\times 1\text{digits} = 9$

$10-99 \quad 90\times 2\text{digits} = 180$

$100-999 \quad 900\times 3\text{digits} = 2700$

$\overline{2889}$

2777^{th} digit is of a 3 digit number

$2889-2777=112=37\times 3+1$

From 999, 37 numbers behind is 962. Its second digit is required answer. So answer is 6.

10. i. $\frac{\text{B in 2011}}{\text{C in 2012}} = \frac{15}{55} = \frac{3}{11}$

ii. Average $= \frac{10+15+40+40}{4} = \frac{105}{4} = 26.25$

iii. Percentage increase in C = $\frac{30\text{-}15}{15}\times 100\% = 100\%$

Section-II: Technical

1. The two balls drawn may be both green, one green and one red or both red.

In these cases, the man receives 40paise, 30paise and 20paise respectively.

Let X be the amount the man receives.

Then

$P[x{=}40] = P[\text{both green}] = \frac{^3C_2}{^5C_2} = 0.3$

$P[x{=}30] = P[\text{one green one red}] = \frac{3C_1\times 2C_1}{5C_2} = 0.6$

$P[x{=}20] = P[\text{both red}] = \frac{2C_2}{5C_2} = 0.1$

$\therefore$ Probability distribution of x is

x	40	30	20		
P(x)	0.3	0.6	0.1		

$E(x) = \Sigma xP(x) = 40\times 0.3 + 30\times 0.6 + 20\times 0.1 = 32\text{paise}$

2. After 1^{st} clock pulse = 1101

After 2^{nd} clock pulse = 1110

:

:

So on after 6^{th} clock pulse =1011

3. $V_o = V\sin\omega t$

$$\frac{dv_o}{dt} = V\omega\cos\omega t$$

$$\text{maximum output} = V\omega = 10\times10^6$$

$$V = \frac{10\times10^6}{2\times\pi\times2\times10^6} = 0.8V$$

4. We will have

$$Y = \overline{\overline{\overline{X}.\overline{Y}}.(1.0) + \overline{\overline{x.y}}.(1.0)}$$

$$= \overline{\overline{(x.y)(1.0)} + \overline{(x.y).(1.0)}}$$

$$= \overline{\overline{(x.y.1.0).(x.y.1.0)}}$$

$$= x.y.1.0 = 0$$

5. $\frac{\partial f}{\partial x} = nx^{n-1};\ \frac{\partial f}{\partial y} = ny^{n-1};\ \frac{\partial f}{\partial z} = nz^{n-1} \Rightarrow \nabla f = n\left[x^{n-1}i + y^{n-1}j + z^{n-1}k\right]$

Since r = xi+yj+zk,

$$\nabla f\cdot r = \left(nx^{n-1}x\right) + \left(ny^{n-1}y\right) + \left(nz^{n-1}z\right) = n\left(x^n + y^n + z^n\right) = nf$$

6. $r_2 = 0.018\Omega/ph \quad x_2 = 0.08\Omega/ph,$

$S_{fl} = 0.04, \quad N_r = N_S(1-s) = 0.96N_S$

at halffull load speed, $\frac{N_r}{2} = 0.48N_S = N_S(1-s)$

$\rightarrow s = 0.52$

$$T_{efl} = \left(\frac{3V^2}{\omega s}\right)\frac{1}{\left[\left(\frac{r_2}{s}\right) + x_2^2\right]}\left(\frac{r_2}{s}\right)$$

$$\left[V_1^2\right]\left[\frac{1}{\left(\frac{0.018}{0.04}\right)^2 + (0.08)^2}\right]\frac{0.018}{0.4}$$

$$= V_2^2\frac{1}{\left[\left(\frac{0.018}{0.52}\right)^2 + (0.08)^2\right]}\frac{0.018}{0.52}$$

$$2.154V_1^2 = 4.55V_2^2$$

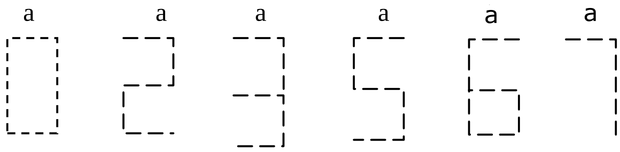
$$1.466V_1 = 2.13V_2$$

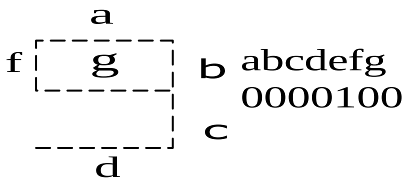
$$\frac{V_1}{V_2} = 1.4526; \frac{V_2}{V_1} = 0.688$$

$$\%\ \text{reduction} = \frac{V_1 - V_2}{V_1} = 1 - \frac{V_2}{V_1} = 1 - 0.688 = 31.15\%$$

7.

x	y	z	a
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

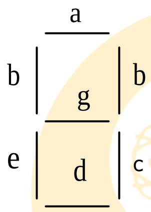




x \ yz	00	01	11	10
0	1		1	1
1		1	1	1

$a = y + xz + x'z'$

A seven-segment decoder with its segment display is shown below:



Segments outputs are given as a, b, c, d, e, f and g

8. $\phi_1 = K\frac{I_1}{2} + K\frac{I_1}{2} = KI_1$

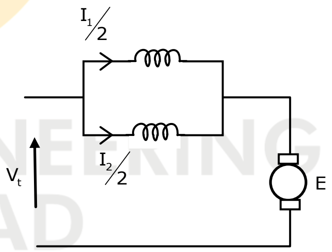
$\phi_2 = KI_2 + KI_2 = 2KI_2$

$T_{e1} = K_a\phi_1 I_1; \quad T_{e2} = K_a\,\phi_2 I_2$

$T_{e1} = T_{e2} \therefore \frac{\phi_1}{\phi_2} = \frac{I_1}{2I_2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow KI_1^2 = 2KI_2^2 \Rightarrow I_1 = \sqrt{2}\, I_2$

$\frac{E_1}{E_2} = \frac{\phi_1\omega_1}{\phi_2\omega_2} \Rightarrow 1 = \frac{1.\omega_1}{\sqrt{2}.\omega_2} \Rightarrow \omega_2 = \frac{\omega_1}{\sqrt{2}}$

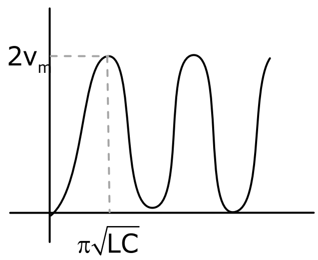


9. $8V = 1600I_D + V_{DS} + 400I_D, \; 8 = 2000I_D + V_{DS}; \; 2000I_D = 8 - V_{DS}$

$I_D = \frac{8}{2000} - \frac{V_{DS}}{2000} = \left(\frac{-1}{2000}\right)V_{DS} + \left(\frac{8}{2000}\right), \; \boxed{m = \frac{-1}{2000}}; \; y = I_D; x = V_{DS}$

10. Restriking Voltage $v(t) = V\left(1 - \cos\frac{t}{\sqrt{LC}}\right)$

$\therefore$ Peak value $= 2V_m$; Time $= \pi\sqrt{LC}$ sec



11. Annual fixed charges = $95 \times 10^5 \times 10\% = 9.5 \times 10^5$

Total running charges = $\left(9 \times 10^5 + 6 \times 10^5\right) = 15 \times 10^5$

Total annual cost = $\left(9.5 \times 10^5 + 15 \times 10^5\right) = 24.5 \times 10^5$

$$\text{Annual load factor} = \frac{\text{No. of units delivered}}{\text{max.demand} \times 8760}$$

$$\therefore \text{No.of units delivered} = 0.5 \times 40000 \times 8760 = 17.52 \times 10^7$$

$$\therefore \text{cost per unit} = \frac{\text{Total annual cost}}{\text{No.of units delivered}}$$

$$= \frac{24.5 \times 10^5}{17.52 \times 10^7} = 100 = 1.398 \text{ paise}$$

12. $$\underset{x\to\infty}{\text{Lt}} \left[\frac{x^2 + 5x + 3}{x^2 + x + 2}\right]^x$$

$$= \underset{x\to\infty}{\text{Lt}} \left(1 + \frac{4x+1}{x^2 + x + 2}\right)^x$$

$$= \underset{x\to\infty}{\text{Lt}} \left[\left(1 + \frac{4x+1}{x^2 + x + 2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{x(4x+1)}{x^2+x+2}}$$

$$= e^{\underset{x\to\infty}{\text{Lt}} \frac{x(4x+1)}{x^2+4x+2}} = e^{\underset{x\to\infty}{\text{Lt}} \frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{1}{x^2}}} = e^4$$

13. $D_s = 0.7788r = 0.233\text{cm}$

$$GMD = D_m = \sqrt[3]{D_{ab} D_{bc} D_{ca}} = 2.29\text{m}$$

$$L = \frac{\mu_o}{2\pi} \ln \frac{D_m}{D_s} = 1.38 \times 10^{-6} \text{H/m} = 1.38 \text{mH/km}$$

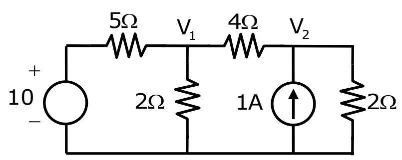
$$L = 1.38 \times 10^{-3} \times 125 = 0.1725\text{H}$$

$$X_L = \omega L = 2 \text{ x } \pi \text{ x } f \text{ x } L = 2 \text{ x } \pi \text{ x } 50 \text{ x } 0.1725 = 54.19\Omega$$

14. Applying KCL at V_1 and V_2

$$\frac{V_1 - 10}{5} + \frac{V_1}{2} + \frac{V_1 - V_2}{4} = 0 \qquad \text{....1}$$

$$\frac{V_2 - V_1}{4} + \frac{V_2}{2} = 1 \qquad \text{....2}$$



By solving (1) and (2), we will get

$V_1 = 2.7\text{V}, \quad V_2 = 2.23\text{V}$

16. $\int_c Mdx + Ndy = \iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) dxdy, M = xy + y^2;\ N = x^2$

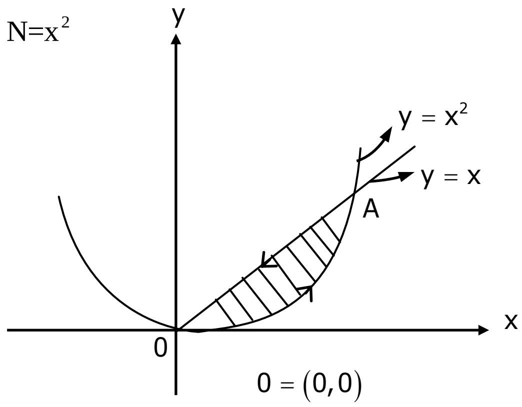
$\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y} = 2x - (x + 2y) = x - 2y$

$\iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) dxdy$

$= \int_{x=0}^{1} \int_{y=x^2}^{x} (x - 2y)\,dydx = \int_{x=0}^{1} \left[xy - y^2\right]_{y=x^2}^{x} dx$

$= \int_{x=0}^{1} \left[(x^2 - x^2) - (x^3 - x^4)\right] dx = \int_{x=0}^{1} (x^4 - x^3)\,dx$

$= \frac{1}{5} - \frac{1}{4} = \frac{-1}{20}$



$0 = (0,0)$
$A = (1,1)$

y varies x^2 to x
x varies 0 to 1

17. Neutral voltage in fault condition

$V_n = 3I_{R0} X_n;\quad I_f = 3I_{R0}$

$I_{R0} = -I_{R1} \frac{x_{2eq}}{x_{0eq} + x_{2eq}} \Rightarrow I_{R1} = \frac{I_{R1}}{x_{1eq} + \frac{x_{2eq} x_{0eq}}{x_{2eq} + x_{0eq}}},\quad I_{R_1} = \frac{1.0}{0.2 + \frac{0.2 \times 0.34}{0.2 \times 0.34}} = 3.068$

$I_{R0} = -(3.068) \times \frac{0.2}{0.2 + 0.34} = 1.136$

$\therefore V_n = 3 \times (1.136) \times (0.05) = 0.17$ PU

$V_n actual = V_{npu} \times V_{base} = 0.17 \times 440K = 75\,kV$

18. The closest scale marking w.r.t to the measured length is chosen. Here it is 231 cm (L). Now with absolute certainty we can say that $L \pm 0.1$ cm. since 0.1 cm is the smallest measurement quantity in this scale.

19. Given $\cos\phi = 0.3 \Rightarrow \phi = 1.266$

$R_p = 2500\Omega;\ L = 20\,mH;\ V = 120V;\ I = 10A$

Power consumed by load, $P_T = VI\cos\phi = 120 \times 10 \times 0.3 = 360W$

$\beta = \tan^{-1}\left(\frac{X_L}{R_p}\right) = \tan^{-1}\left(\frac{2\pi fL}{R_p}\right)$

$= \tan^{-1}\left(\frac{2\pi \times 50 \times 20 \times 10^{-3}}{2500}\right) = 0.00251$ rad

Actual wattmeter reading = $= [1 + \tan\phi \tan\beta]$ true power

$= [1 + \tan(72.54°)\tan(0.143°)]360 = 362.87\,w$

Power loss, $P_L = \frac{V^2}{R_p} = \frac{(120)^2}{2500} = 5.76W$

Total wattmeter reading = $362.84 + 5.76 = 368.63W$

$\therefore \%Error = \frac{P_w - P_T}{P_T} \times 100 = \frac{368.63 - 360}{360} \times 100 = 2.397\%$

20. We have, $e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} .....$

$\sin\frac{1}{z} = \frac{1}{z} - \frac{1}{3!z^3} + ...$

There is an essential singularity at z = 0

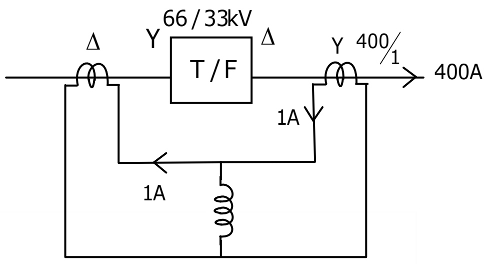
The residue at z = 0 is coefficient of $\frac{1}{z}$ in Laurent series of integrand, which is 1

So $\oint_{|z|=1} e^{1/z} \sin\frac{1}{z} dz = 2\pi i$

21. $I_1 = I_2 K$

$I_1 = \frac{400}{\sqrt{3}} \times \frac{33}{\frac{66}{\sqrt{3}}} = 200A \qquad i_1 = \frac{1}{\sqrt{3}} A$

$\therefore$ HV side CT ratio $\frac{I_1}{i_1} = \frac{200}{1/\sqrt{3}} = 200\sqrt{3}$



22. In position 1, R_1 is in shunt with $R_2 + R_3 + R_m$

$\therefore I_1 R_1 = I_m (R_2 + R_2 + R_3)$

Given $I_1 = 10A; \; I_m = 1mA; \; R_m = 50\Omega$

$\therefore 10 R_1 = 1\times 10^{-3} [R_2 + R_3 + 50]$

$\Rightarrow R_1 = 10^{-4} [R_2 + R_3 + 50]$

In position 2, $R_1 + R_2$ is in shunt with $R_3 + R_m$

$I_2 (R_1 + R_2) = I_m (R_3 + R_m)$

$\uparrow$

5A $5(R_1 + R_2) = I_m (R_3 + R_m)$

$5(R_1 + R_2) = 1\times 10^{-3} [R_3 + 50]$

$\therefore R_1 + R_2 = 1\times 10^{-3} [R_3 + 50] \Rightarrow R_1 + R_2 = 2\times 10^{-4} [R_3 + 50]$

In position 3, $R_1 + R_2 + R_3$ is in shunt with R_m

$I_3 (R_1 + R_2 + R_3) = I_m R_m$

$\downarrow$

1A $\therefore R_1 + R_2 + R_3 = 1\times 10^{-3} \times 50 = 0.05$

$R_1 + R_2 + R_3 = 0.05$

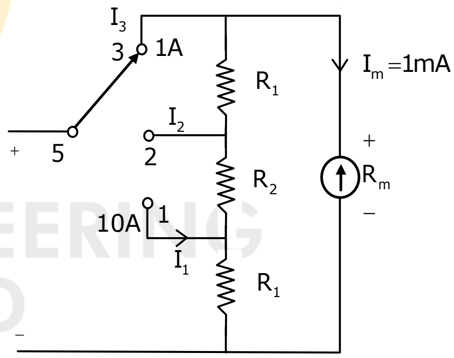
$R_1 + R_2 = 2\times 10^{-4} [R_3 + 50] \Rightarrow R_3 = 0.0399\Omega$

$R_1 + R_2 = 0.01, \; R_2 = 0.01 - R_1$

$R_1 = 10^{-4} (R_2 + R_3 + 50)$

$R_1 = 10^{-4} [0.01 - R_1 + 0.0399 + 50]$

$\therefore R_1 = 0.005\Omega, \; R_2 = 0.005\Omega$



23. Given $R_1 = 4.5K\Omega;\ C_1 = 1\mu F$

$R_2 = 6.5K\Omega;\ R_3 = 650\Omega;\ \omega = 1000 \text{ rad} \times \sec$

$$R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1+\omega^2 R_1^2 C_1^2} = \frac{(1000)^2 \times (4.5\times10^3)\times(1\times10^{-6})^2 \times (6.5\times10^3)\times(650)}{1+(1000)^2 (4.5\times10^3)(1\times10^{-6})^2} = 894.7\Omega$$

24. $L_{eq} = 4+8+6-(2\times2)+(2\times4) = 22H$

25. $\omega = 0,\ V_{out} = V_{in}$

$\omega = \infty,\ V_{out} = 0$

It represents low pass filter

26. $I_o = 6A$

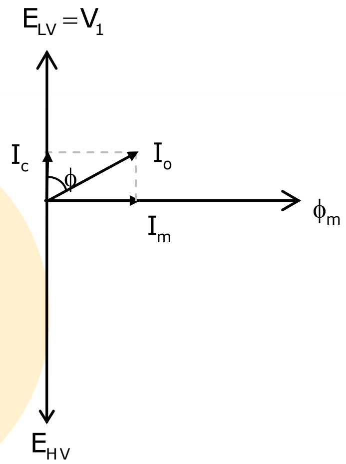
The low voltage side of transformer is excited.

So, care-loss component of no load current

$$I_C = \frac{250}{200} = 1.25A$$

Magnetizing component $I_m = \sqrt{I_o^2 - I_c^2} = 5.87A$

No-load p.f. $= \cos\phi = \frac{I_c}{I_o} = 0.208$



27.

Q(t)	K	G	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Q(t) \ KG	00	01	11	10
0			1	
1	1	1	1	

$Q(t+1) = KG + Qk'$

28. $Z_x = 0.009 + j0.04 = 0.041\angle 77.31$

$Z_y = 0.004 + j0.02 = 0.0203\angle 78.69°$

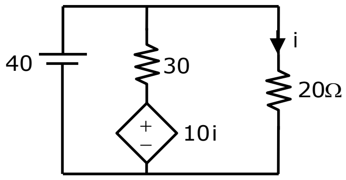
$$(KVA)_x = \frac{Z_y}{Z_x + Z_y} \times 600\angle -31.78°$$

$$= \frac{0.0203\angle 78.69°}{0.041\angle 77.31° + 0.0203\angle 78.69°} \times 600\angle -31.78°$$

$= 199.7\angle -30.85;\ \text{p.f} = 0.85(\text{lagging})$

$$(KVA)_y = \frac{Z_x}{Z_x + Z_y} \times 600\angle -31.78° = 401.37\angle -32.236;\ \text{p.f} = 0.845\ (\text{lagging})$$

29. $t=0^{-}$ $\therefore i(0)=\frac{40}{20}=2A$



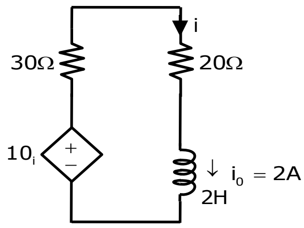
at $t=0^{+}$ the circuit becomes

$10\,i(t)=30i(t)+20i(t)+2\frac{di}{dt}$

$2\frac{di}{dt}+40\,i(t)=0$

$i(t)=k\,e^{-20t}$

$i(t)=2e^{-20t}$ $\quad i(0)=2A$



30. $V_0=-\left(\frac{R}{R}\right)-\left(\frac{R}{R}\right)V_1=V_1$